## A CRITERIAL GENERALIZATION OF THE CHARACTERISTICS OF VORTEX- <br> STABILIZED PLASMA GENERATORS

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The present paper is a continuation of a previously published paper [1] on the criterial generalization of the current-voltage and thermal characteristics of vortex-stabilized plasma generators.

In accordance with [1], the thermal efficiency, the dimensionless arc voltage, and the mean mass enthalpy of the stream (in what follows these quantities are denoted by the letter $\psi$ ) in geometrically similar single-chamber plasma generators without external magnetic fields or interelectrode inserts are determined to the first approximation by the criteria

$$
\begin{gather*}
K_{1}=\frac{c_{1} I^{2}}{G d}, \quad R=\frac{c_{2} G}{d}, \\
K=\frac{c_{3}}{p d},  \tag{1}\\
\psi=\psi\left(\frac{c_{1} I^{2}}{G d}, \frac{c_{2} G}{d}, \frac{c_{3}}{p d}\right) . \tag{2}
\end{gather*}
$$

Here $c_{1}, c_{2}, c_{3}$ are dimensional constants, $I$ is the current, $G$ is the gas flow rate, $d$ is the internal diameter of the arc chamber, $p$ is a characteristic pressure (in what follows it is the static pressure in the stream at the end of the arc chamber).


Fig. 1. Current voltage-characteristics of the arc in a single-chamber plasma generator. Working medium-air, polarity-reversed. Point $1-\mathrm{d}=1$ $\mathrm{cm}, G=12 \mathrm{~g} \cdot \mathrm{sec}^{-1}, 2-\mathrm{d}=2 \mathrm{~cm}, G=12 \mathrm{~g}$. $\sec ^{-1}, 3-d=3.5 \mathrm{~cm}, G=12 \mathrm{~g} \cdot \mathrm{sec}^{-1}, 4-\mathrm{d}=$ $=2 \mathrm{~cm}, G=24 \mathrm{~g} \cdot \mathrm{sec}^{-1}, 5-\mathrm{d}=3.5 \mathrm{~cm}, G=$ $=24 \mathrm{~g} \cdot \sec ^{-1}$, The broken curves (1), (2), (3) correspond to $\mathrm{d}=3.5,2$ and 1 cm for $\mathrm{G}=12 \mathrm{~g}$. $\sec ^{-1}$ and are given by formula (7).

The criteria [2] form part of a complete system of criteria [1] and by a series of simplifying assumptions may be reduced to a common form by means of suitable transformations.

Since any power combination of criteria is also a criterion, we may give them different forms, but in this case the number of criteria describing a given process should remain constant. The power combinations obtained in this manner may be simpler or more convenient for processing specific experimental material, although from the point of view of the theory of similarity such combining does not give a criterion which is new in principle. Thus, for example, we may obtain from (1)

$$
K_{1}^{0.5} R^{0.5}=\frac{c_{4} I_{1}}{d}, \quad K_{1}^{0.5} R^{-0.5} K=\frac{c_{5} I}{G p d}, \quad K_{1}^{0.5} R^{-0.5}=\frac{c_{6} I}{G}
$$

If instead of $\mathrm{c}_{1}{ }^{2} / \mathrm{Gd}$ we take the simpler combination $\mathrm{c}_{4} \mathrm{I} / \mathrm{d}$, then we shall have the system of criteria
in place of (1).
Then discarding dimensional constants in the criteria and in the dimensionless arc voltage, we may represent the generalized cursentvoltage characteristic in the form

$$
\begin{equation*}
\frac{U_{g} d}{I}=f\left(\frac{I}{d}, \frac{G}{d}, p d\right) \tag{4}
\end{equation*}
$$

where $U_{g}$ is the arc voltage.
The dimensional complex $I / d$ is well known and is widely applied in generalizing the current-voltage characteristics of cylindrical arcs [1. 3]. Thus, as one would expect, dimensional complexes determining the similarity of cylindrical arcs may be obtained from the similarity criteria of arcs in a gas stream by eliminating $G$.

Generalizations [4] and a series of formulas [1] for $\mathrm{U}_{\mathrm{g}}$ are obtained allowing for only one dimensional complex $I^{2} / G d$ in (2). Such generalizations are valid within a narrow range of variation of $R$ and $K$. Generalizations in the form

$$
\begin{equation*}
\frac{U_{g} d}{I}=A\left(\frac{l^{2}}{G d}\right)^{-a}\left(\frac{G}{d}\right)^{-\beta} \quad(A=\text { const }) \tag{5}
\end{equation*}
$$

(cf. [1]) widen the region of applicability of the generalized functions and indicate the dependence of $\mathrm{U}_{\mathrm{g}}$ on d . However, the dependence of $\mathrm{U}_{\mathrm{g}}$ on I and d is more complicated for a series of gases, and formulas of the type (5) with constant exponents are also insufficient to take into account all the features of the current-voltage characteristics. Figure 1 gives current-voltage characteristics for various values of the arc chamber diameter and air flow rate which display these features. It is clear that for small currents the quantity $U_{g}$ increases as $d$ increases, while for large currents we note a tendency for $U_{g}$ to decrease as $d$ increases. Similar behavior of the arc is also observed in the case of hydrogen [5]. Thus the exponents of $I$ and $d$ in the formulas should be variables.


Fig. 2. Current -voltage characteristics of an arc in a single-chamber plasma generator. Working medium-air, polarity-reversed; $i=I / d A \cdot \mathrm{~cm}^{-1}, u=U_{g} d / I V \cdot \mathrm{~cm}$. - $\mathrm{A}^{-1}, \mathrm{p}=10 \mathrm{~N} \cdot \mathrm{~cm}^{-2}$; the points correspond to the values: $1-\mathrm{d}=3.5 \mathrm{~cm}, G=8 \mathrm{~g} \cdot \mathrm{sec}^{-1}, 2-\mathrm{d}=3.5 \mathrm{~cm}$, $G=12 \mathrm{~g} \cdot \mathrm{sec}^{-1}, 3-\mathrm{d}=3.5 \mathrm{~cm}, G=24 \mathrm{~g} \cdot \mathrm{sec}^{-1}, 4-$ $-\mathrm{d}=2 \mathrm{~cm}, G=8 \mathrm{~g} \cdot \mathrm{sec}^{-1}, 5-\mathrm{d}=1 \mathrm{~cm}, G=4.1$

$$
\mathrm{g} \cdot \mathrm{sec}^{-1} ; 6-\mathrm{d}=1 \mathrm{~cm}, G=12 \mathrm{~g} \cdot \mathrm{sec}^{-1}
$$

Figure 2 gives graphs of the dependence of $u=U_{g} d / I$ on $i=I / d$ for various values of $\mathrm{g} / \mathrm{d}$ and pd , the stratification with respect to $\mathrm{G} / \mathrm{d}$ and pd being quite clear. Stratification of the same order also accompanies generalization using the complexes $I^{2} / G d, G / d, p d$. Neglect of this stratification may lead to a large error in calculating $U_{g}$. It is
clear that for constant $G / d$ and $p d, \lg u$ does not depend linearly on $\lg i$ for air. If we describe this dependence by a second-order curve in some range of variation of the parameters

$$
\lg u=c-\left(\alpha_{0}-\alpha \lg i\right) \lg i \quad\left(c=\text { const }, \alpha_{0}=\text { const }, \alpha=\text { const }\right)
$$

and regard $U_{g} d / I$ as a power function of $G / d$ and pd with constant exponents $B$ and $\gamma$, we obtain a formula of the type

$$
\begin{equation*}
\lg u=\lg A-\left(\alpha_{0}-\alpha \lg i\right)+\beta \lg (G / d)+\gamma \lg (p d) \tag{6}
\end{equation*}
$$

Processing of the data of Fig. 2 in form ( 6 ) for the arc voltages in a single-chamber plasma generator with reverse polarity leads to the following expression:

$$
\begin{gather*}
U_{\mathrm{g}}=1.3 \cdot 10^{4} G^{0.3} d^{-0.26} p^{0.04} i^{0.245} \mid \mathrm{g} \cdot-1.312 \mathrm{~b} \\
10<i<10^{2.4} \mathrm{~A} \cdot \mathrm{~cm}^{-1} \\
2<G / d<12 \mathrm{gram} \cdot \mathrm{sec}^{-1} \cdot \mathrm{~cm}^{-1}  \tag{7}\\
6<p d<40 \mathrm{~N} \cdot \mathrm{~cm}^{-1} \\
0.6<d<3.5 \mathrm{~cm} \\
30<I<200 \mathrm{~A}
\end{gather*}
$$

The continuous lines in Fig. 1 show the values of $U_{g}$ determined by (7). They are close to the experimental curves.

We note one more important fact connected with the structure of formula (6). When the dimensionless arc voltage $u_{g}$ is expressed by a power function of the determining criteria

$$
U_{g}=A\left(c_{4} i\right)^{\alpha} R^{3} K^{\gamma}
$$

the upper limit of burning stability for the arc is expressed by the formula

$$
U_{\mathrm{gmax}}=\operatorname{const} U_{\theta}
$$

assuming that the supply voltage $\mathrm{U}_{\mathrm{b}}$ is independent of the current, i.e. the above limit is independent of the size of the current which, generally speaking, is contradicted by experiment.

It is not difficult to show that the upper boundary of stable arc burning, determined by the relation between $\mathrm{U}_{\mathrm{g}}$ and I according to formula (6) and having the form

$$
U_{g \max }=U_{b}\left(1+\alpha_{0}-2 x \lg 0\right)^{-1}
$$

is a curve increasing with the current, which agrees quantitatively and qualitatively with the experimental data [6].

Thus allowing for the nonlinear dependence of $\lg u$ on $\lg$ i enables us to describe a series of features of the current-voltage characteristics.

The direction in which $p$ influences $\mathrm{U}_{\mathrm{g}}$ in formula (7) agrees with the data of [?]. However, this influence is weak, and further experiments embracing a variation of $p$ of at least two orders of magnitude are necessary to make the numerical value of the exponent more exact.

Numerous experiments have shown that the current-voltage characteristics are reproducible with an accuracy of $\pm 10 \%$. In our experiments the arc in a new arc chamber with clean surfaces had a greater voltage.


Fig. 3. Schematic of a two-chamber plasma generaror. 1) Internal electrode,
2) outlet electrode, 3) arc, $G_{1}$ and $G_{2}$ gas inlets.
As the state of the surface changed as a result of arc burning, the voltage usually decreased. Evidently this is explained by the formation of irregularities on the surface of the chamber, creating more favorable conditions for shorting, and consequently for a reduction of the average length of the arc. If we allow for this fact, then the maximum meansquare deviation of formula (7) from the experimental data, equal to $10 \%$ in the range of variation of the parameters indicated, may be regarded as satisfactory. Since the coefficients $A, \alpha_{0}, \alpha, \beta, \gamma$ themselves depend on the similarity criteria, they must henceforth be determined over a wider range of variation of the parameters.

Together with the current-voltage characteristics the thermal efficiency was determined. This is the ratio of the increase of stagnation
enthalpy of the gas in the plasma generator to the arc power. Processing of the data for reverse polarity led to the formula

$$
\begin{equation*}
\eta_{-}=-1+\frac{2.52}{(P d)^{0.05}}-0.17 \lg \frac{I^{2}}{G d}, \quad \frac{l}{d}=30 \tag{8}
\end{equation*}
$$

where $l$ is the length of the arc chamber, and the range of variation of the remaining parameters is indicated below formula (7). In the experiments cited, the ratio $l / \mathrm{d}$ was chosen on the grounds that the cathode spot should be located inside the arc chamber for minimal current $I_{\text {min }}$ and maximal flow rate $G_{\text {max }}$. Since the length of the arc decreases in the range indicated as I increases and $G$ decreases, for $I>I_{\min }$ and $G<G_{\max }$ the maximum possible value of the efficiency (which is attained at smaller values of $1 / \mathrm{d}$ ) is somewhat larger than that obtained from (8). Experiments have shown that within the limits of measurement error the magnitudes of the thermal efficiencies coincide for positive and negative polarities in one and the same plasma generator. This may be explained by the fact that the increase in losses due to the higher temperature in the case of negative polarity (since $\mathrm{U}_{\mathrm{g}}$ is higher than for forward polarity) is compensated by the increase in arc length as compared with forward polarity.


Fig. 4. Generalized current-voltage characteristics of an arc in a two-chamber plasma generator. Working medium argon, polarity-forward, $p=10 \mathrm{~N} \cdot \mathrm{~cm}^{-2}$, the points correspond to the values: $d_{2}=0.8 \mathrm{~cm}, G=6.7 \mathrm{~g} \cdot \mathrm{sec}^{-1}, \mathrm{I}=$ $80-280 \mathrm{~A}, 1-\mathrm{d}_{2}=1 \mathrm{~cm}, G=6.6 \mathrm{~g} \cdot \mathrm{sec}^{-1}, \quad i=88-$ $-200 \mathrm{~A}, 2-\mathrm{d}_{2}=1.6 \mathrm{~cm}, G=14 \mathrm{~g} \cdot \mathrm{sec}^{-1}, I=150-270$ $\mathrm{A} ; 3-\mathrm{d}_{2}=5 \mathrm{~cm}, G=105 \mathrm{~g} \cdot \mathrm{sec}^{-1}, \mathrm{I}=560-2200 \mathrm{~A}$, $4-d_{2}=5 \mathrm{~cm}, \quad G=160-165 \mathrm{~g} \cdot \mathrm{sec}^{-1}, \quad \mathrm{l}=800-2600 \mathrm{~A}$; $5-\mathrm{d}_{2}=5 \mathrm{~cm}, G=124-130 \mathrm{~g} \cdot \mathrm{sec}^{-1}, \mathrm{I}=590-2300 \mathrm{~A}$.

The straight line corresponds to formula (11).
For given $I, G$, $d$ the time average of the mean mass specific stagnation enthalpy of the gas at the outlet of the plasma generator $h$ (for a gas temperature at the inlet of about $290^{\circ} \mathrm{K}$ ) may be deter mined from the equation of conservation of energy

$$
\begin{equation*}
U_{g} I \eta_{-}+G h_{0}=G h \tag{9}
\end{equation*}
$$

where $h_{0}$ is the specific stagnation enthalpy of the gas at the inlet ${ }^{\text {a }}$ and $\mathrm{U}_{\mathrm{g}}$ and $\eta$ - are determined from (7) and (8).

Formula (7) and the well-known stability conditions allow us to determine the necessary supply voltage and the magnitude of the ballast resistance. Thus the characteristics of a one-chamber plasma generator and its power supply may be evaluated on the basis of the empirical formulas obtained. The working regime of a two-chamber plasma generator, apart from current, gas flow rate and geometrical dimensions, also depends on the relation of the flow rates through the individual chambers, i. e., on $g=G_{2} / G$, where $G=G_{1}+G_{2}$ (see Fig. 3). Thus for such geometrically similar plasma generators the criterial equations should be written in the following form

$$
\begin{equation*}
\psi=\psi\left(\frac{c_{4} I}{d_{3}}, \frac{c_{2} G}{d_{3}}, \frac{c_{3}}{p d_{2}}, \frac{G_{2}}{G}\right) \tag{10}
\end{equation*}
$$

Systematic measurements were carried out to determine the possibility of generalizing the current-voltage characteristics in the form (10) to include two-chamber plasma generators. The following ratios
were obtained when working with argon: $\mathrm{d}_{1} / \mathrm{d}_{2}=1.4, l_{1} / \mathrm{d}_{1}=6, l_{2} / \mathrm{d}_{2}=$ $=16$. Here $l_{1}$ and $l_{2}$ are the lengths of electrodes 1 and 2 , respectively. The experiments were carried out at constant $g(g=0.77-0.80)$, and so an equation of the form (4) may be employed to generalize the data in a particular case of this type. On being processed the data revealed marked stratification with respect to the values of $\mathrm{G} / \mathrm{d}_{2}$ and $\mathrm{pd}_{2}$ of lg i as a function of $\lg u$. If we neglect the second-order properties of the current-voltage characteristics for this case, the formula obtained for the arc voltage in argon with forward polarity has the form

$$
\begin{gathered}
U_{g}=180 I^{-0.23} G^{0.33} d_{2}^{0.30} \mathrm{~V} \\
90<i<500 \mathrm{~A} \cdot \mathrm{~cm}^{-1}, \quad 7<G / d_{2}<33 \mathrm{~g} \cdot \mathrm{sec}^{-1} \cdot \mathrm{~cm}^{-1}, ~(11) \\
0.8<d_{2}<5 \mathrm{~cm}, \quad p=10 \mathrm{~N} \cdot \mathrm{~cm}^{-2}, 50<I<2600 \mathrm{~A}_{1}
\end{gathered}
$$

Figure 4 compares formula (11) with experiment, where $\varphi=$ $=I^{-0.23} G^{0.33} d_{2}^{0.30}$, and the continuous curve is determined from (11). It is clear from the graph that the maximum departure of the experimental points from formula (11) does not exceed $\pm 15 \%$. This confirms the conclusion that it is possible to make a criterial generalization of the current-voltage characteristics of the arc in a two-chamber plasma generator within a wide range of variation of current, flow rate and chamber dimensions.

The material outlined above shows that at the present level of knowledge of the processes in one- and two-chamber plasma generators
of the vortex type, the method of generalized characteristics is an effective means of estimating the design parameters.

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